Investigation of a parameter and force reconstruction technique using non-linear image data

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Introduction

We are interested in exploring the process of a natural structure conforming to its environment. Interesting examples of such naturally occurring conformations are gossamer/thin membrane structures and biological systems. These are highly deformable and often delicate structures, which must be studied in their natural conditions using methods such as elastography, optics and magnetic resonance imaging.

With the current generation of computational tools we are able to begin characterizing and modeling these complex and highly non-linear systems. These problems do not lend themselves to traditional measurement techniques. There is a pressing need for new measurement technologies designed for this type of problem. Application of inverse and hybrid methods to this task is currently an area of active research. Identification problems are notoriously difficult because they are highly ill-conditioned and traditionally have had to be defined analytically. This has put limits on the complexity of the simulations and generally has limited the solutions to linear response.

Some general features of these types of systems can be abstracted:

1. large deflections,
2. large rotations,
3. large strains,
4. geometric complexity,
5. time dependence,
6. non-linear material and plasticity,
7. bulky structure and
8. "natural" loading such as gravity, wind etc..

We designed a test with a sub-set of these features to focus on some aspects of the problem[1]. Figure 1 shows our choice of a simplified system. Our model experiences large deflections, large rotations, geometric complexity, non-linear material and a bulky structure. The cantilevered condition allows large deflections and rotations, the holes introduce geometric complexity, the structure is made of a natural gum rubber and gravity provides the primary load. This is not a model that is representative of a particular application, but one that provides an instance of the abstract features described above.

Experimental Results

The specimen is shown in Figure 1. It is 38 cm long and its cross-section is 5×2.5 cm^2. It is fabricated from a commercial natural gum rubber sold as a sheet. The 2.5 cm diameter holes were drilled 5 and 18 cm from the fixed end.

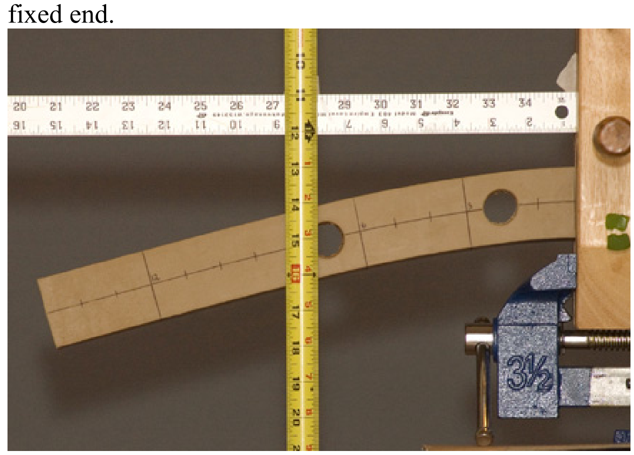

Figure 1. Test Specimen.

Parameter Identification

We chose to use a Mooney Rivlin modeling of the rubber material properties: c0 and α. We determined that for the range of strain exercised in this experimental set-up α did not have an effect. Therefore we needed only to identify the material parameter c0. The units of c0 are Pa. We used a cantilever set up of the model as shown in Figure 1 to retrieve displacement data (measured by digitizing the photograph). We studied the sensitivity of the parameter identification of c0 to:

a) the location of data points, and
b) the number of data points.

Table 1 lists the results using two data points. The mean is 257,658 and the standard deviation is 929. More data points resulted in very similar mean values of c0 but an improved standard deviation.

We determined the value of c0 to be 258,235 Pa using 13 data points. Thus we conclude that a reasonably robust identification is achieved with two or more data points.

Table 1. Results for parameter identification.

Data Set (1 pair) (see Fig. 2)	c0, Pa
10,15	257,886
5,15	257,796
5,10	256,221
8,13	257,583
12,15	258,803
ū	257,658
σ	929

Force Identification

The force identification problem is shown in Figure 2. The structure is oriented in its stiff configuration and the end of the cantilever is supported on a small scale. The structure is weighted at mid-length and the scale measured the reaction force R as 3.0 N.

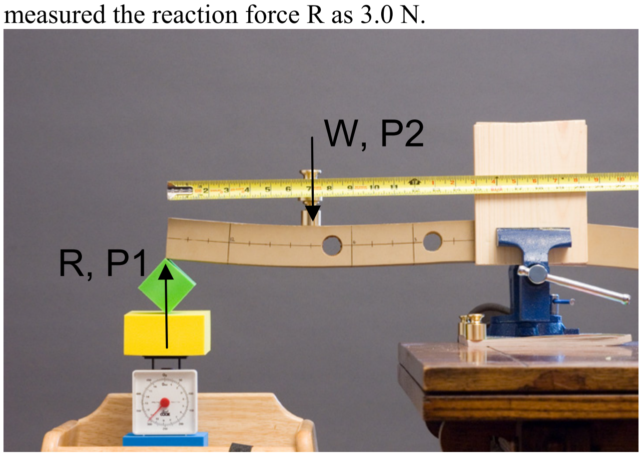

Figure 2. Illustration of load case.

These results are given in Table 2. We reconstructed P1 with a mean of 3.0 N and a standard deviation of 0.3. We reconstructed P2 with a mean of 3.0 and a standard deviation of 0.7. Using 15 data points we reconstructed the measured values closely: P1=2.9 N and P2=2.9 N. We conclude that P2 is especially sensitive to the location of data points used as input. Even using 15 data points, the reconstructed forces are different from the experimentally measured values by about the same amount in both load cases. We conclude that we may be seeing the results of ill-conditioning.

Table 2. Results for force identification.

	Reconstructed Force	
Data Set	P1 (R)	P2 (W)
10,15	2.727	-2.277
5,15	3.130	-3.417
5,10	3.509	-4.177
8,13	2.907	-2.748
12,15	2.567	-1.824
ū	2.968	-2.888
σ	0.368	0.931
Compare:	3.1	-3.0

Conclusion

It was our purpose to fully specify an analytical model of a natural material conforming to its environment. To set up controlled parameters within which to work, we designed a rubber specimen to manifest certain of the characteristics of naturally conforming systems. We focused on identifying and defining the quality of two parameters: material properties and multiple forces.

We now have the material and force information we need to fully specify a finite element model. We can now leverage the power of a commercial finite element package. Now any combination of loads or post-processing can be performed on the fully specified model. Not only can we reconstruct stresses and strains for the load cases used in the identification, we can also create new load situations with reasonable confidence.

Following on from this work, there are several opportunities for exploration. The small but definite difference in the c0 reconstruction for the different orientations is interesting phenomenon and one that could use further explanation. We would like to find an arrangement of the structure that reduces the sensitivity of the reconstructions to error. We are also interested in continuing to modify our experiment so as to include other attributes of naturally conforming systems. We will thereby gain insight into other aspects of this challenging problem and the potential of inverse methods to provide a reliable method of fully specifying them.

References

[1] Doyle, J.F., *Modern Experimental Stress Analysis: completing the solution of partially specified problems,* Wiley & Sons, UK, 2004.